EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS
Earthquake Engng. Struct. Dyn. **28**, 143–156 (1999)

ACTIVE CONTROL OF MDOF STRUCTURES WITH SUPPLEMENTAL ELECTRORHEOLOGICAL FLUID DAMPERS

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SUMMARY

A method for design of an active control system for multistorey structures using Electrorheological (ER) dampers is presented. Incorporated at various levels of a structural frame, ER dampers are used to improve the response of the structure during earthquakes. Optimal control theory was used to design the ER devices. The aim of the design is to find the most suitable combination of the minimum required forces produced by the ER dampers to obtain the optimal structural response. The mechanical response of ER fluid dampers is regulated by an electric field, depending on the displacements and velocities of the frame. Numerical analysis of an ER damped seven-storey structure is represented as an example. Significant improvement of the structural response was obtained using optimal active controlled ER dampers compared to passive controlled and uncontrolled structures. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: active control; electrorheological fluid dampers; optimal control theory

INTRODUCTION

Passive energy dissipating systems such as viscous dampers, tuned mass dampers and base isolation systems have been installed in existing buildings resulting in improved structural response to earthquakes. Such systems have inherent limitations, however. For example they are generally tuned to the first mode vibration. Active ER dampers can be effective over a much wider frequency range. Hence the design of an active controlled ER damped structure is a logical extension of the passive control systems.

ER dampers are commonly used for vibration control of mechanical systems.[1] The resulting damping force developed by the ER device depends on the physical properties of the fluid used, the pattern of flow in the damper and its size. When an electric field is applied, the behaviour of the electrorheological fluid is nearly viscoplastic and the shear stress in it has to exceed the developed 'yield' stress to initiate flow. This mechanism is responsible for the controllable viscoplastic behaviour of ER dampers. The magnitude value of the resulting damping force for ER dampers is considerably large, making them suitable for structural engineering applications.

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CCC 0098–8847/99/020143–14$17·50

Received 14 February 1998
Revised 23 July 1998

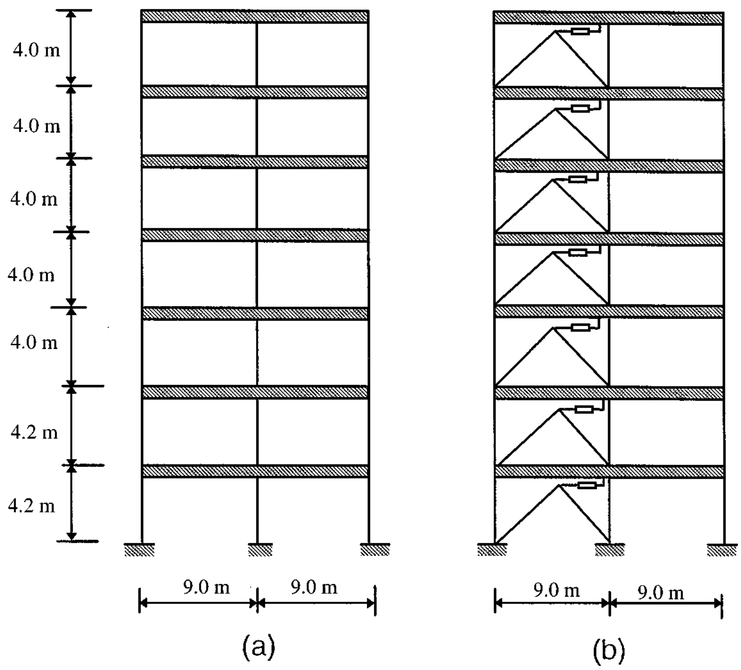

Figure 1. Seven-storey structure: (a) uncontrolled; (b) ER damped

This work assumes that active controlled ER dampers are placed between chevron braces and the rigid floor diaphragm on each level of the structure (Figure 1). An optimal linear passive control approach[2] will be used to determine the viscous constants of the ER devices. Optimal control forces at every time step will then be determined using active control theory. These forces are in practice obtained by varying the applied electric field. The design process developed in this paper can be used for the design of new structures or for retrofitting and rehabilitation.

PASSIVE CONTROL THEORY

The response of a structure provided with supplemental dissipating devices is described by the following dynamic equation of equilibrium:[3]

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Lf_{\mathrm{e}}(t) + Du(t) \tag{1}$$

where M, C, K are the mass, damping, and stiffness matrices, respectively; $x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$ are the displacement, velocity and acceleration vectors, u is the vector of forces in the supplemental devices, f_{e} is the external excitation, D and L are the control and excitation force matrices, respectively.

The force produced by a linear fluid viscous device, u_{v}, is proportional to the velocity of the displacement of the device piston (up to a limiting frequency, beyond which the device becomes

viscoelastic):[4]

$$u_{\mathrm{v}} = C_{\mathrm{d}}\dot{x}(t) \tag{2}$$

where C_{d} is the viscous characteristic of the ER device, $\dot{x}(t)$ is the velocity of the piston in the fluid viscous medium.

The resulting damping force in an ER device is

$$u_{\mathrm{ER}}(t) = C_{\mathrm{d}}\dot{x}(t) + F\,\mathrm{sign}[\dot{x}(t)] \tag{3}$$

where F is the controllable yield force.

The system of second-order differential equations (1) may be simplified by transformation into the space-state form

$$\dot{z}(t) = Az(t) + Bu(t) + Hf_{\mathrm{e}}(t) \tag{4}$$

where $z(t) = [x(t), \dot{x}(t)]^{\mathrm{T}}$ is the $2n$ state-space vector of the displacements and velocities of the structure;

$$A_{2n\times 2n} = \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \tag{5}$$

is the system matrix, and

$$B_{2n\times m} = \begin{bmatrix} 0 \\ M^{-1}D \end{bmatrix} \quad \text{and} \quad H_{2n\times r} = \begin{bmatrix} 0 \\ M^{-1}L \end{bmatrix} \tag{6}$$

are and location matrices specifying, respectively, the locations of controllers and external excitations in state space.

For the case when the control forces are linear, they can be written as

$$u(t) = Gz(t) = [G_x, G_{\dot{x}}]z(t) = G_x x(t) + G_{\dot{x}}\dot{x}(t) \tag{7}$$

where G is the gain matrix.

The equation of motion reduces to[2]

$$\dot{z}(t) = A_{\mathrm{c}}z(t) + Hf(t) \tag{8}$$

where the matrix of the controlled system, A_{c}, is

$$A_{\mathrm{c}} = A + BG \tag{9}$$

The matrix G can be determined from the minimization of the performance index

$$J = \int_0^T [z^{\mathrm{T}}(t)Qz(t) + u_{\mathrm{c}}^{\mathrm{T}}(t)Ru_{\mathrm{c}}(t)]\,\mathrm{d}t \tag{10}$$

or, using equation (7),

$$J = \int_0^T \{[z^{\mathrm{T}}(t)Q + G^{\mathrm{T}}RG]z(t)\}\,\mathrm{d}t \tag{11}$$

where

$$G = -0{\cdot}5R^{-1}B^{\mathrm{T}}P \tag{12}$$

where P is the solution of Ricatti algebric equation:

$$A^{\mathrm{T}}P + PA - 0{\cdot}5PBR^{-1}B^{\mathrm{T}}P + 2Q = 0 \tag{13}$$

Q is a $2n \times 2n$ positive-semi-definite matrix, and R is an $m \times m$ positive-definite matrix.

The matrices Q and R are weighting matrices, whose magnitudes are assigned according to the relative importance attached to the state variables and to the control forces in the minimization procedure (see Reference 3). For this study these matrices were used in a parametric range:

$$R = 10^{-p} I_{n \times n}; \qquad Q = I_{2n \times 2n} \tag{13}$$

where I is a unit diagonal matrix, and p is a parameter which used to adjust the solution's weights to lie in the practical range.

The control forces are obtained using equation (7) in the following form:

$$\begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{Bmatrix} = \begin{bmatrix} g_{11,x} & g_{12,x} & \cdots & g_{1n,x} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1,x} & g_{n2,x} & \cdots & g_{nn,x} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} + \begin{bmatrix} g_{11,\dot{x}} & g_{12,\dot{x}} & \cdots & g_{1n,\dot{x}} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1,\dot{x}} & g_{n2,\dot{x}} & \cdots & g_{nn,\dot{x}} \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{Bmatrix} \tag{14}$$

For the structures provided by linear viscous devices the control forces are dependent on constant stiffness and the damping coefficients of the devices[2]

$$\begin{aligned} \begin{Bmatrix} u_{V1} \\ u_{V2} \\ \vdots \\ u_{Vn} \end{Bmatrix} &= \begin{bmatrix} k_1 & -k_2 & & & \\ -k_2 & k_1 + k_2 & -k_3 & & \\ \ddots & \ddots & \ddots & \ddots & -k_n \\ & & & -k_n & k_n \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \\ &+ \begin{bmatrix} c_1 & -c_2 & & & \\ -c_2 & c_1 + c_2 & -c_3 & & \\ \ddots & \ddots & \ddots & \ddots & -c_n \\ & & & -c_n & c_n \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{Bmatrix} \end{aligned} \tag{15}$$

The coefficients c_{ij} and k_{ij} are obtained from the gain coefficients $g_{ij,x}$ and $g_{ij,\dot{x}}$ using approximations, for example, storey-drift formulation. The relationship between storey drifts and storey displacements is obtained using transformation

$$x(t) = \theta d(t) \tag{16}$$

where

$$\theta = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ & 1 & 1 & & 1 \\ & & & \cdots & 1 \\ & & & \ddots & \vdots \\ & & & & 1 \end{bmatrix} \tag{17}$$

The control forces in the terms of storey drifts and drift velocities are

$$v(t) = \theta^{\mathrm{T}} u(t) \tag{18}$$

or, using equation (14)

$$v(t) = G_{\mathrm{d}} d(t) + G_{\dot{\mathrm{d}}} \dot{d}(t) \tag{19}$$

where

$$G_{\mathrm{d}} = \theta^{\mathrm{T}} G_x \theta, \qquad G_{\dot{\mathrm{d}}} = \theta^{\mathrm{T}} G_{\dot{x}} \theta \tag{20}$$

For the case when the control forces are supplied by the braces, the brace forces can be written[2]

$$v^*(t) = K_{\mathrm{d}} d(t) + C_{\mathrm{d}} \dot{d}(t) \tag{21}$$

in which

$$K_{\mathrm{d}} = \theta^{\mathrm{T}} K_x \theta = \mathrm{diag}(\Delta k_i) \tag{22}$$

and

$$C_{\mathrm{d}} = \theta^{\mathrm{T}} C_{\dot{x}} \theta = \mathrm{diag}(\Delta c_i) \tag{23}$$

where Δk_i and Δc_i are the supplemental stiffness and damping from each brace at the level i.

The stiffness and damping can be assumed independent. The least-squares approximation applied to the difference between equations (18) and (21) results in the explicit:[2]

$$\frac{\mathrm{d}}{\mathrm{d}\dot{d}_i(t)} \left\{ \int_0^{t_{\mathrm{f}}} \sum_j \left[g_{ij,\dot{\mathrm{d}}} \dot{d}_j(t) - \Delta c_i \dot{d}_i(t) \right]^2 \mathrm{d}t \right\} = 0 \tag{24}$$

and leads to

$$\Delta c_i = \frac{\int_0^{t_{\mathrm{f}}} \sum_j g_{ij,\dot{\mathrm{d}}} \dot{d}_j(t)\, \mathrm{d}t}{\int_0^{t_{\mathrm{f}}} \dot{d}_i(t)\, \mathrm{d}t} \tag{25}$$

where t_{f} is the total time for the event.

If only one mode is relevant for the structural response, equation (25) may be further simplified:

$$\Delta c_i = \frac{\sum_j g_{ij,\dot{\mathrm{d}}} \Phi_{jm}}{\Phi_{im}} \tag{26}$$

where Φ_{im} is the element of the eigenvector corresponding to mode m and degree of freedom i.

The damping coefficients are not dependent on the earthquake history, but only on the characteristics of the structure. Therefore, equation (26) may be used to obtain the viscous characteristics of the ER dampers.

ACTIVE CONTROL STRATEGY

The response of the structure can be further improved by controlling it actively. This provides more truly optimal control because the excitation term is not ignored in the derivation of the Riccati matrix $P(t)$, as is the case using Passive Control Theory (PCT). The history of the external excitation at any time, which is available up to that time instant, can be utilized to improve the behaviour of the building.

The form of the performance index usually chosen for study is time-dependent and defined by the equation[3]

$$J(t) = z^{\mathrm{T}}(t) Q z(t) + u^{\mathrm{T}}(t) R u(t) \tag{27}$$

Considering the evolution of the state vector $z(t)$ over a small time interval Δt and assuming that the open-loop system matrix A possesses distinct eigenvalues, the system of equations (4) can be written in state-space representation:[3]

$$z(t) = T y(t) \tag{28}$$

where T is $2n \times 2n$ modal matrix whose columns are eigenvectors of A.

Following Soong,[3] the decoupled state-space equation governing $y(t)$ has the form, upon substituting equation (28) into equation (4),

$$\dot{y}(t) = \Lambda y(t) + q(t) \tag{29}$$

where

$$\Lambda = T^{-1} A T \tag{30}$$

is a diagonal matrix whose diagonal elements are the complex eigenvalues of the matrix A, and

$$q(t) = T^{-1} [Bu(t) + H f_{\mathrm{e}}(t)] \tag{31}$$

Over a small time interval Δt, the vector $y(t)$ becomes

$$\begin{aligned} y(t) &= \int_0^{t-\Delta t} \exp[\Lambda(t-\tau)] q(\tau)\, \mathrm{d}\tau + \int_{t-\Delta t}^{t} \exp[\Lambda(t-\tau)] q(\tau)\, \mathrm{d}\tau \\ &\cong \exp \Lambda\, \Delta t \times y(t-\Delta t) + \frac{\Delta t}{2} [\exp \Lambda \Delta t q(t-\Delta t) + q(t)] \end{aligned} \tag{32}$$

For the state vector $z(t)$, equations (28), (31) and (32) lead to

$$z(t) = T d(t-\Delta t) + \frac{\Delta t}{2} [B u_{\mathrm{c}}(t) + H f_{\mathrm{e}}(t)] \tag{33}$$

where

$$d(t-\Delta t) = \exp(\Lambda\, \Delta t) T^{-1} \left\{ z(t-\Delta t) + \frac{\Delta t}{2} [B u_{\mathrm{c}}(t-\Delta t) + H f_{\mathrm{e}}(t-\Delta t)] \right\} \tag{34}$$

Considering closed-loop instantaneous control,[3]

$$u_{\mathrm{c}}(t) = -\frac{\Delta t}{2} R^{-1} B^{\mathrm{T}} Q z(t) \tag{35}$$

where the response state vector $z(t)$ is, following equations (33) and (35),

$$z(t) = \left[I + \frac{\Delta t^2}{4} B R^{-1} B^{\mathrm{T}} Q \right]^{-1} \left[T d(t-\Delta t) + \frac{\Delta t}{2} H f_{\mathrm{e}}(t) \right] \tag{36}$$

We see that using the presented design technique the solution of the Riccati equation is not required. This means that the control design procedure is very simple.

By analysing the building's response during the earthquake, optimal forces at every structural level will be obtained for every time increment. The electric field in each device at every time step will be varied in such a way that the forces produced in the dissipating devices will be equal to those obtained by the optimization procedure. For every device the linear viscous force can be obtained using equation (2). If the optimal force is less than the viscous part then no electric field is needed. And if the optimal force is greater than the viscous part, then the value of the electric

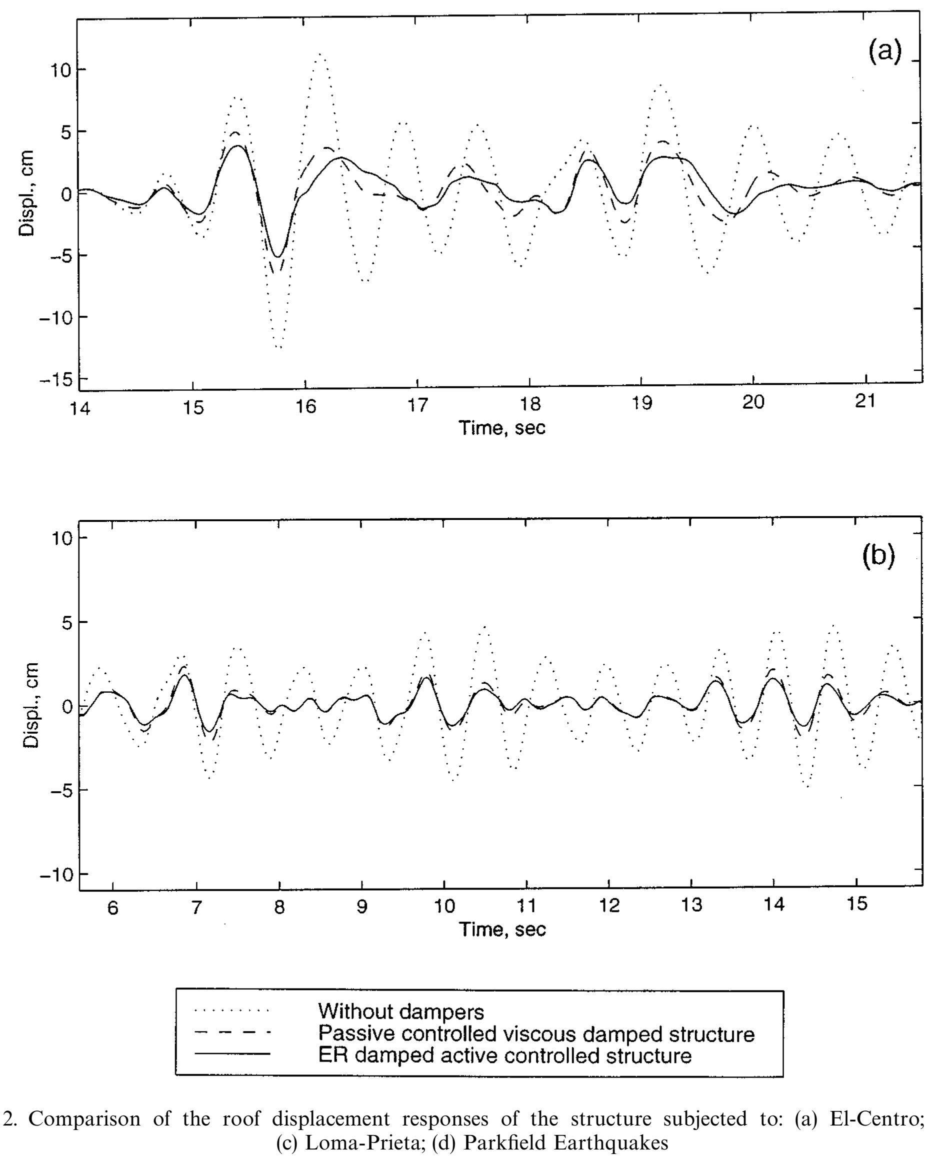

Figure 2. Comparison of the roof displacement responses of the structure subjected to: (a) El-Centro; (b) Taft; (c) Loma-Prieta; (d) Parkfield Earthquakes

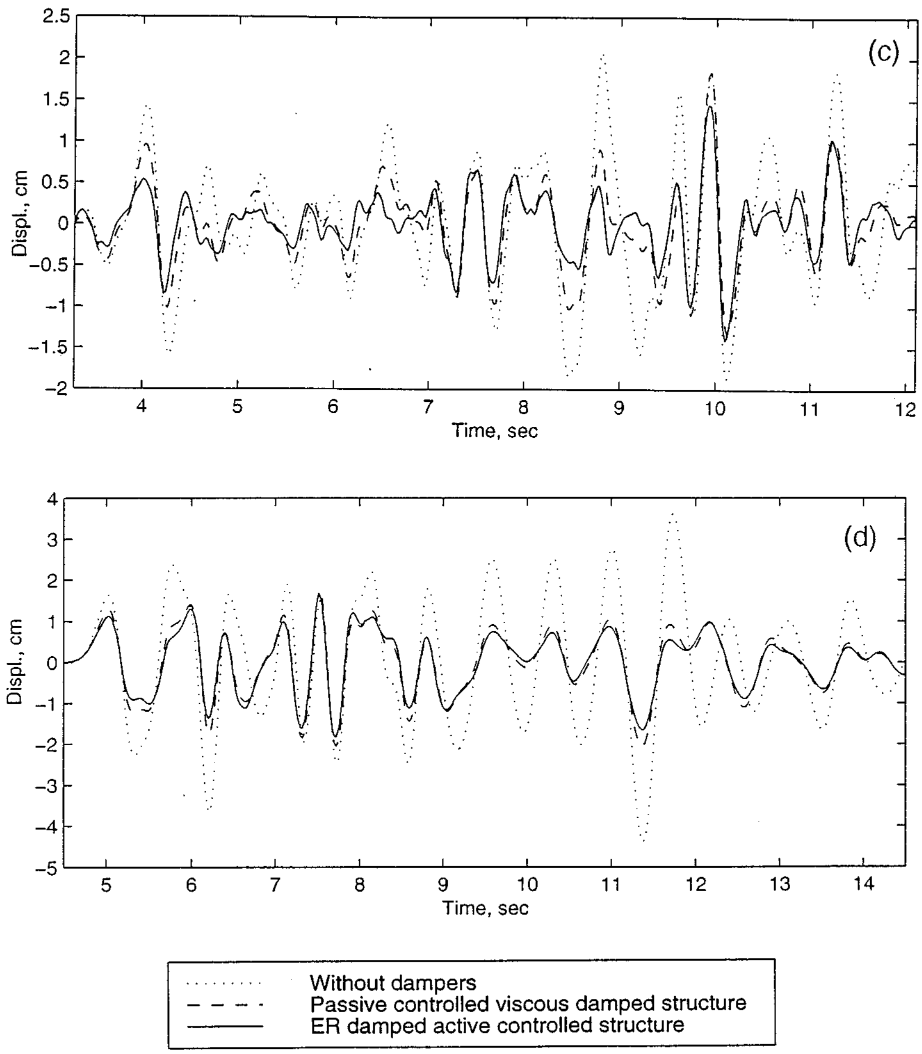

Figure 2. (*Continued*)

field produces in the ER device a supplemental force equal to the difference between the optimal value and the viscous part.

NUMERICAL EXAMPLE

To investigate the effectiveness of the proposed design technique, simulations were carried out for a seven-storey building. A shear framed structure, with stiff beams was chosen (Figure 1). Responses were computed for four different seismic excitations (specified below). All simulations were performed using routines written in MATLAB. The structure was characterized by the

following matrices:

$$M = 8{\cdot}75 \times 10^4 I_{7\times 7}\ (\text{kg mass})$$

$$K = \begin{bmatrix} 29{\cdot}28 & -14{\cdot}64 & & & & & 0 \\ -14{\cdot}64 & 31{\cdot}59 & -16{\cdot}95 & & & & \\ & -16{\cdot}95 & 30{\cdot}96 & -14{\cdot}01 & & & \\ & & -14{\cdot}01 & 28{\cdot}02 & -14{\cdot}01 & & \\ & & & -14{\cdot}01 & 25{\cdot}13 & -11{\cdot}12 & \\ & & & & -11{\cdot}12 & 22{\cdot}24 & -11{\cdot}12 \\ & & & & & -11{\cdot}12 & 11{\cdot}12 \end{bmatrix} \times 10^7\ (\text{N/m})$$

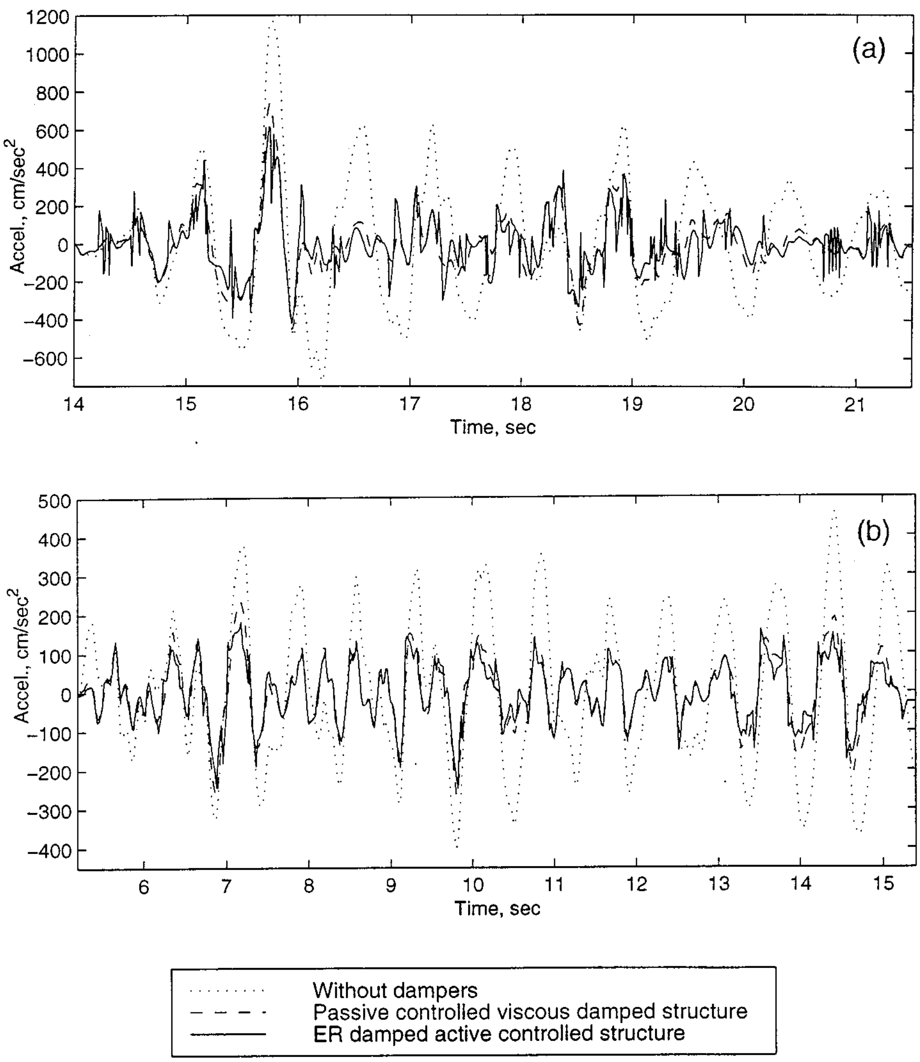

Figure 3. Roof accelerations time history for the structure subjected to: (a) El-Centro; (b) Taft; (c) Loma-Prieta; (d) Parkfield Earthquakes

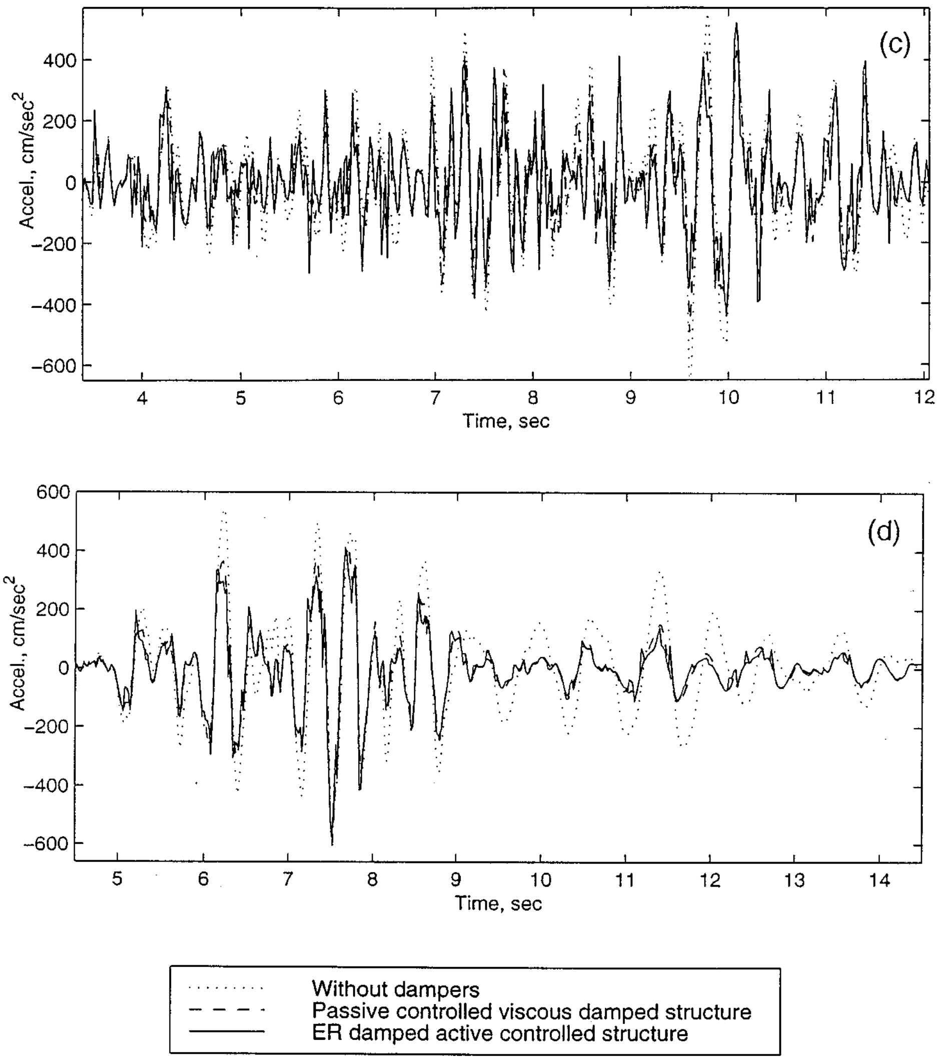

Figure 3. (*Continued*)

where M is the mass matrix of the structure, I is a unit diagonal matrix, and K is the structural stiffness matrix.

An initial damping ratio of 1 per cent was assumed for the uncontrolled structure. The PCT analysis was used to obtain the optimal viscous characteristics of the devices at all levels of the structure. The optimization was carried out using the technique of Gluck *et al.*[2]

(implemented in a MATLAB routine). The optimal values of the viscous coefficients were found to be

$$\Delta C_k = \begin{bmatrix} 1{\cdot}7219 & & & & & & 0 \\ & 1{\cdot}7219 & & & & & \\ & & 1{\cdot}9936 & & & & \\ & & & 1{\cdot}6478 & & & \\ & & & & 1{\cdot}6478 & & \\ & & & & & 1{\cdot}3079 & \\ & & & & & & 1{\cdot}3079 \end{bmatrix} \times 10^6\,(N \times \mathrm{sec})/\mathrm{m}$$

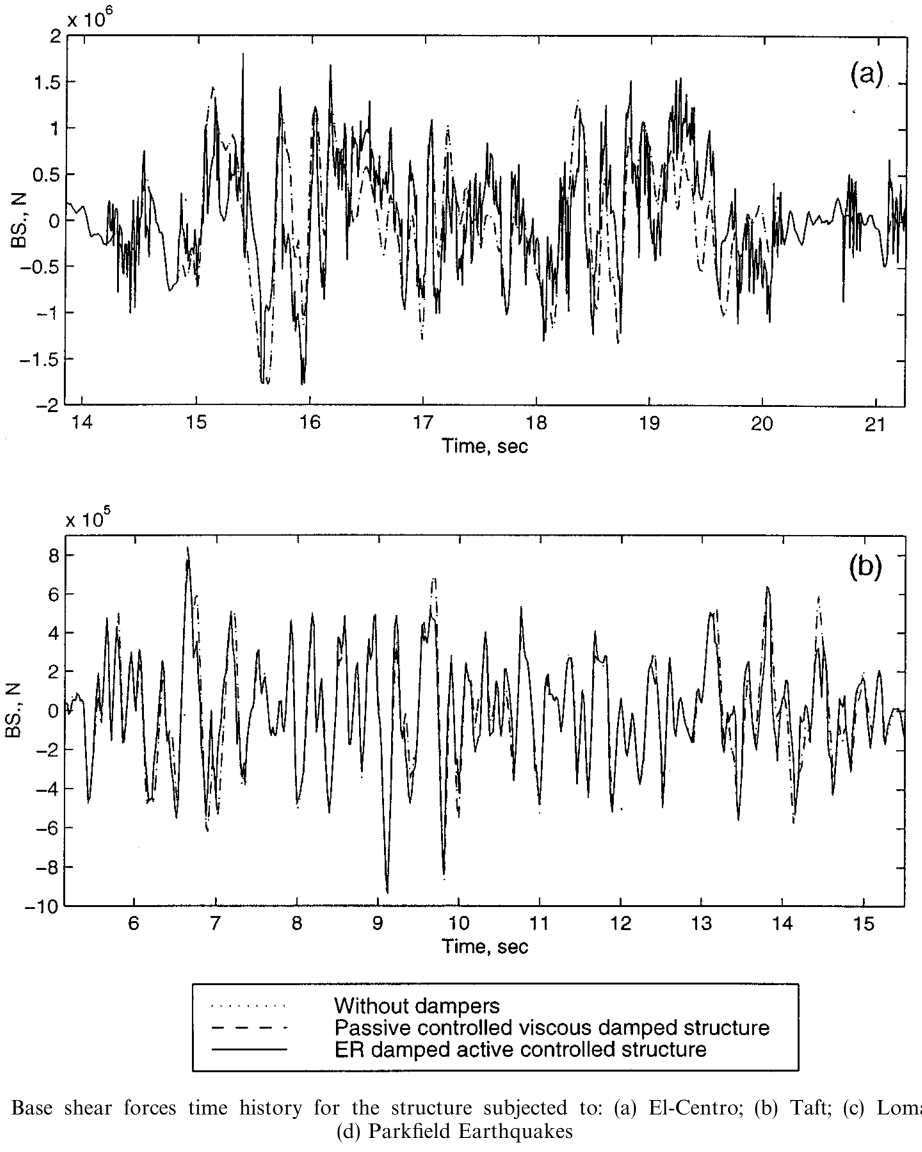

Figure 4. Base shear forces time history for the structure subjected to: (a) El-Centro; (b) Taft; (c) Loma-Prieta; (d) Parkfield Earthquakes

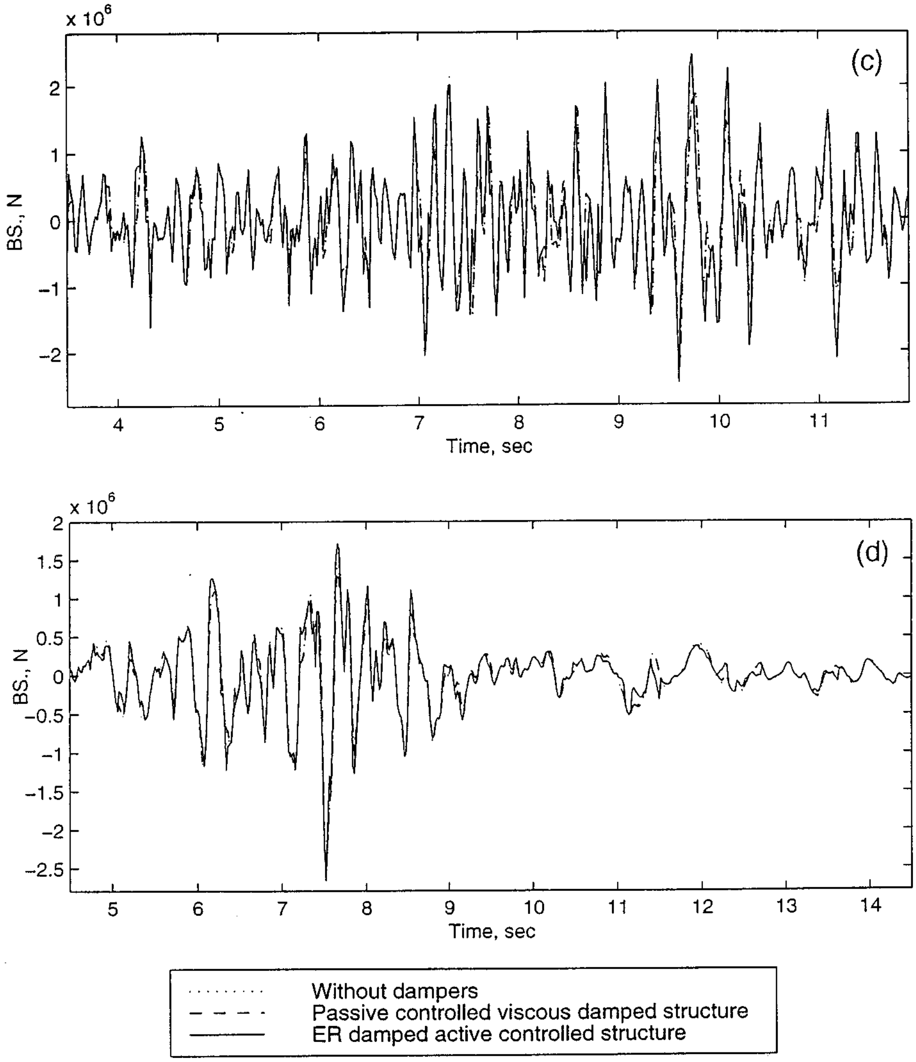

Figure 4. (*Continued*)

Based on these results, we used devices with $C = 1{\cdot}07 \times 10^6$ N $\times$ sec/m for the 6th and 7th floors, and $C = 1{\cdot}6 \times 10^6$ N $\times$ sec/m for floors 1–5.

The following four seismic excitations were used as input in the analysis:

El-Centro S00E, 1940,
Taft N21E, 1952,
Loma-Prieta N90E, 1989, and
Parkfield N85E.

The roof displacement, roof acceleration and base shear force time histories of the structure are shown in Figures 2, 3 and 4, respectively. Peak displacements and base shear forces

Table I. Response of the uncontrolled structure to different seismic loads

	Storey number	El-Centro	Taft	Loma-Prieta	Parkfield
x (cm)	7	12·85	5·10	2·08	4·39
	6	12·07	4·79	1·88	4·14
	5	10·56	4·19	1·57	3·65
	4	8·83	3·52	1·28	3·10
	3	6·72	2·70	1·00	2·40
	2	4·76	1·93	0·77	1·74
	1	2·45	0·98	0·45	0·89
BS (N)		1 752 900	926 430	2 459 700	2 626 500

Table II. Response of the optimally designed viscous damped passive controlled structure

	Storey number	El-Centro	Taft	Loma-Prieta	Parkfield
x (cm)	7	6·86	2·31	1·84	2·05
	6	6·45	2·16	1·63	1·93
	5	5·72	1·89	1·34	1·72
	4	4·90	1·60	1·15	1·48
	3	3·86	1·25	1·01	1·16
	2	2·83	0·92	0·81	·085
	1	1·49	0·48	0·48	0·46
BS (N)		1 776 200	934 610	2 436 200	2 570 500

Table III. Response of the optimally designed active controlled structure with ER dampers

	Storey number	El-Centro	Taft	Loma-Prieta	Parkfield
x (cm)	7	5·31	1·81	1·45	1·81
	6	5·14	1·72	1·42	1·78
	5	4·84	1·55	1·28	1·63
	4	3·76	1·35	1·09	1·45
	3	2·92	1·09	0·97	1·25
	2	2·20	0·84	0·82	1·02
	1	1·10	0·45	0·44	0·55
BS (N)		1 802 300	934 560	2 470 300	2 666 200

for the uncontrolled, the viscous damped passive controlled and the ER damped active controlled structure, respectively, are presented in Tables I–III.

The passive controlled viscous damped structure has for the case presented, a peak displacement reduction up to 55 per cent (see Tables I and II). For the active controlled structure

reduction in the peak displacements of up to 65 per cent were achieved (see Tables I and III). There was no significant increase in the peak values of accelerations and of base shear compared to the uncontrolled structure when either passive or active controlled devices were simulated.

CONCLUSIONS

A procedure was developed for optimal design of active controlled ER damped structures. Viscous properties of the ER devices were selected using PCT, and ACT was then used to find the optimal control forces.

Numerical simulation showed reductions in peak displacements of a seven-storey active controlled structure of up to 65 per cent without increases in base shear forces and accelerations. Hence, active controlled fluid ER dampers can be expected to significantly improve the behaviour of structures during earthquakes.

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